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NAVAL AIR DEVELOPMENT CENTER

WARMINSTER, PA. 18974

SYSTEMS DIRECTORATE

TECHNICAL MEMORANDUM 2-82 26 JANUARY 1982

ON MATHEMATICAL PROGRAMS FOR OPTIMIZING COMBAT AIR PATROL STATIONING AND INTERCEPT VECTORING TACTICS

DEPARTMENT OF THE NAVY
NAVAL AIR DEVELOPMENT CENTER
WARMINSTER, PA. 18974

Systems Directorate

TECHNICAL MEMORANDUM 2-82 26 January 1982

On Mathematical Programs for Optimizing Combat Air Patrol Stationing and Intercept Vectoring Tactics

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CONTENTS

FIGURES

TABLES

1. INTRODUCTION

1.1 Purpose

This technical memorandum discusses mathematical programming formulations for the problem of optimizing the stationing and vectoring of Combat Air Patrol (CAP) aircraft employed in the air defense of naval task forces.

1.2 Background

Previous analyses (see references (a) and (b)) are based on certain simplifying assumptions. For example, they assume that CAP aircraft always fly lead collision courses with the target. There is no assurance that this tactic is always optimal for CAP aircraft carrying long-range air-to-air missiles (AAMs). This document suggests that some of the assumptions made in previous studies can be weakened considerably if the problem is formulated as a mathematical program.

1.3 Scope

In this memorandum only the case where all aircraft and missiles move in straight lines and at constant speed is considered. It is assumed that the threat moves directly toward the center of the Task Force (TF), and that CAP aircraft may or may not be vectored to the target by friendly airborne or other long-range surveillance and air defense early warning systems. All action is treated as taking place in a Euclidean plane.

1.4 Approach

Consider only one type of scenario, which is based on the disposition of forces shown in Figure 1. In this scenario, a CAP aircraft is stationed at a distance from TF center given by the CAP station radius. A threat

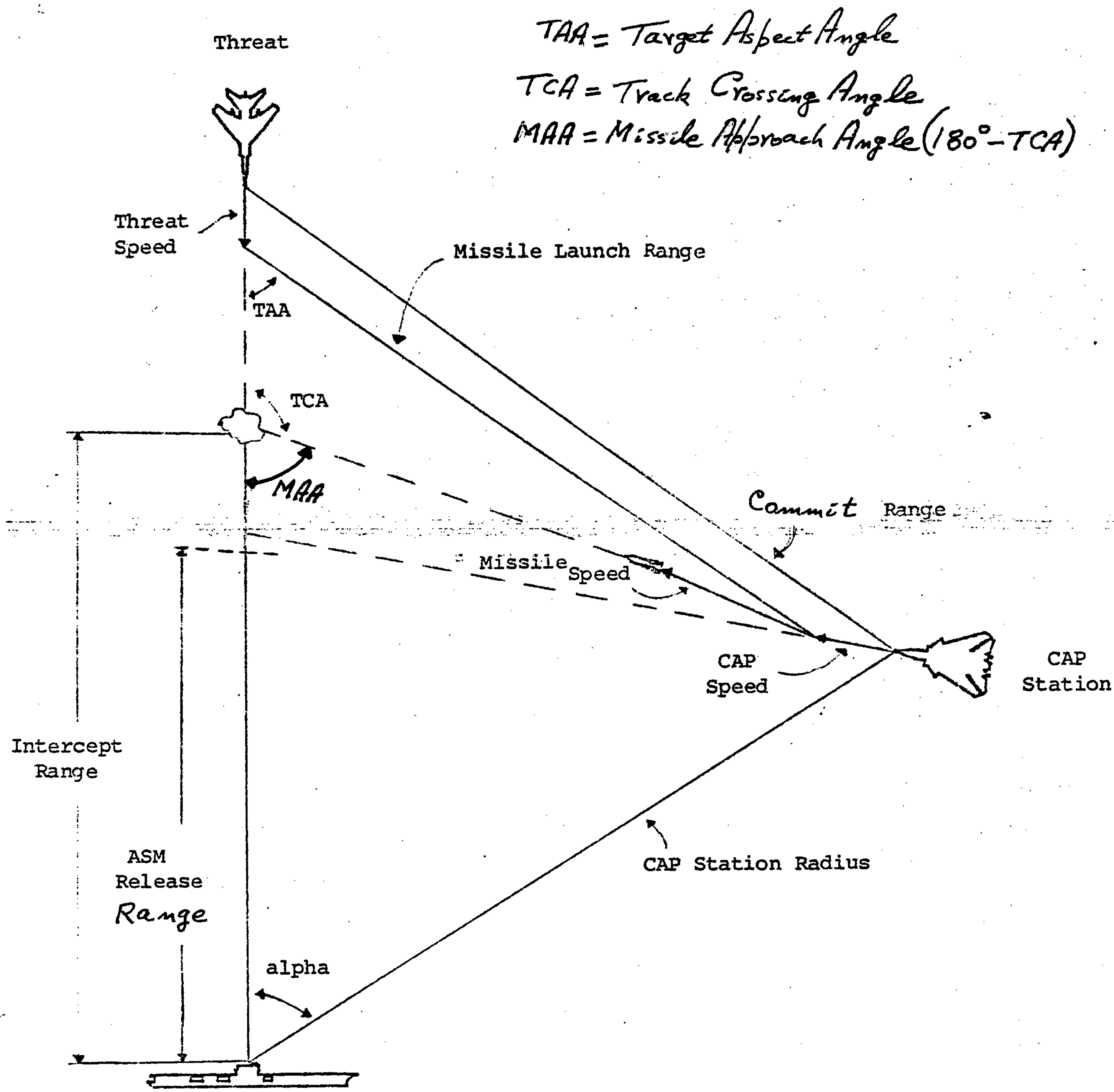

Figure 1. Tactical Scenario

aircraft group heads directly for TF center along a radial making an angle alpha with the CAP station's radial. At some point in its approach, the threat is detected and (after some delay for threat analysis and command decision) the CAP aircraft are vectored to engage the threat. After they come within range of the threat aircraft group, the CAP aircraft launch their AAMs. The AAMs intercept the threat at some Intercept Range as illustrated in Figure 1. To be considered successful, the intercept must occur before the threat aircraft reach their anti-ship missile (ASM) launch range (i.e., the Intercept Range must exceed the ASM Release Range shown in Figure 1).

There are a number of limitations on CAP and AAM capability that must be honored. For example, it is not practical to station CAP aircraft too far from TF center, AAMs may have to be launched at certain target aspect angles (or so as to attain certain aspect angles at intercept), and so forth. Section 3 gives a detailed discussion and mathematical development of the constraints considered in this paper.

The maximum sector coverage provided by a CAP stationing and vectoring policy can be identified with the maximum angle alpha (shown in Figure 1) that gives an intercept before the threat aircraft reach their ASM Release Range and is compatible with all the constraints on CAP and AAM capabilities. Other measures of effectiveness are conceivable and some of them are discussed in Section 4.

1.5 Results

The results are a mathematical programming formulation for a family of CAP stationing and intercept vectoring problems. A large variety of particular cases can be obtained by relaxing selected constraints. Unfortunately, the resulting mathematical programming problems are nonlinear, and so can be solved only by numerical methods. However, computer routines for solving such mathematical problems are available and could be applied to CAP stationing and vectoring problems of the type developed here (see, for example, reference (c)). Some suggestions on ways to formulate the problems to facilitate their solution are offered in Section 5.

2. SCENARIO AND DEFINITION OF VARIABLES

2.1 Scenario

The reader is referred to Figure 2 for a sketch illustrating the relations among the key variables selected for describing the problem in mathematical form. In this figure, O is the TF center and S is the CAP stationing point. The threat approaches TF center along the radial O-T and at a speed VT. TD is the distance of the threat from TF center when it is first detected by elements of the TF -- which may or may not be the CAP aircraft themselves. RhoD is the distance of the threat from the CAP when the threat is detected, and omegaD is the target aspect angle of the threat as viewed from the CAP at that time. Lower-case h is time that elapses between detection of the threat and commitment of the CAP to attack it. TC is the distance of the threat from TF center when the CAP are committed; rhoC and omegaC are the distance of the threat from the CAP and the aspect angle of the threat as viewed from the CAP at that time. L is the point at which AAMs are launched from the CAP aircraft, and rhoF is the distance of that point from the CAP station point. OmegaF is the angle as shown on Figure 2. VF is the speed (assumed constant) of the CAP aircraft while flying to the AAM launch point and tF is the time required for that flight. RhoL is the distance of the CAP aircraft from the threat at the time the AAMs are launched and omegaL is the aspect angle of the threat as viewed from the CAP aircraft at AAM launch. TM is the distance of the threat from TF center when intercept occurs. OmegaM is the aspect angle of the threat as viewed from the missile at that time. VM is the speed of the missile (assumed constant) during its flight, and LM is its time of flight from launch to intercept. Angles are taken to be positive in the directions indicated in Figure 2. Angles omegaL, omegaM and omegaF may vary from -180° to +180°. The other angles (alpha, omegaD and omegaC) may vary from 0° to +180°.

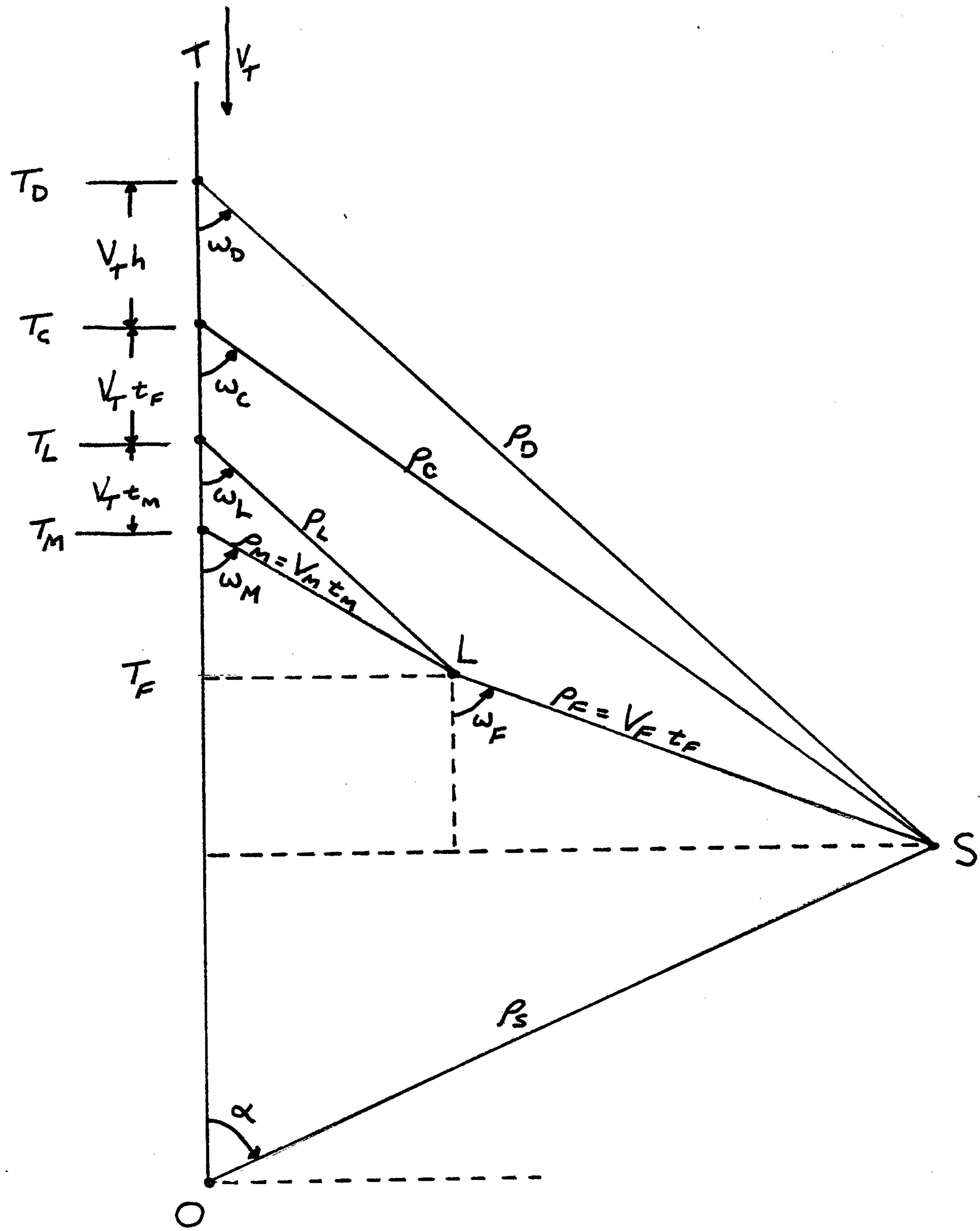

Figure 2. Labeled Schematic

2.2 Relations Among the Variables

Many of the relations among the variables presented in Table 1 are easily derived from trigonometric formulas for right-angled triangles. The others follow from the assumed constant speed of the threat, the CAP aircraft, and the AAMs.

2.2.1 Discussion of the Relations Among the Variables

One of the relations (3), (4) and (5) shown in Table 1 is redundant and can be eliminated.

This memorandum considers only those cases in which the following parameters have known fixed non-negative values given by the conditions of the problem: VF, VM, and VT.

The parameters shown in Table 2 are considered to be unknowns controllable by the TF's anti-air warfare system, subject to the relations among them given in Table 1 and to appropriate constraints to be discussed in Section 3. Because of the relations among them and to the constraints, not all of the controllable variables can be adjusted independently.

TABLE 1. RELATIONS AMONG THE VARIABLES

$$\rho_D \sin \omega_D = \rho_S \sin \alpha \tag{1}$$

$$\rho_C \sin \omega_C = \rho_S \sin \alpha \tag{2}$$

$$\rho_L \sin \omega_L = \rho_M \sin \omega_M \tag{3}$$

$$\rho_F \sin \omega_F + \rho_L \sin \omega_L = \rho_S \sin \alpha \tag{4}$$

$$\rho_F \sin \omega_F + \rho_M \sin \omega_M = \rho_S \sin \alpha \tag{5}$$

$$\rho_D \cos \omega_D = T_D - \rho_S \cos \alpha \tag{6}$$

$$\rho_C \cos \omega_C = T_C - \rho_S \cos \alpha \tag{7}$$

$$\rho_L \cos \omega_L + \rho_F \cos \omega_F = T_L - \rho_S \cos \alpha \tag{8}$$

$$\rho_M \cos \omega_M + \rho_F \cos \omega_F = T_M - \rho_S \cos \alpha \tag{9}$$

$$\rho_F \cos \omega_F = T_F - \rho_S \cos \alpha \tag{10}$$

$$T_C = T_D - V_T h \tag{11}$$

$$T_L = T_C - V_T t_F \tag{12}$$

$$T_M = T_L - V_T t_M \tag{13}$$

$$T_F = T_M - \rho_M \cos \omega_M \tag{14}$$

$$\rho_F = V_F t_F \tag{15}$$

$$\rho_M = V_M t_M \tag{16}$$

TABLE 2. CONTROLLABLE VARIABLES

ρ_S	α	
ρ_D	ω_D	T_D
ρ_C	ω_C	T_C
ρ_F (t_F)	ω_F	T_F
ρ_L	ω_L	T_L
ρ_M (t_M)	ω_M	T_M

3. CONSTRAINTS

3.1 Forced Constraints

The forced constraints that seem most reasonable to impose are shown in Table 3. Others may be appropriate in different contexts. Not all of them will actually be effective in any given problem situation. Those that are not to be effective can either be omitted, or else their bounds can be taken so large (or so small) that the constraint has no effect on the solution.

One interpretation of the forced constraints will be offered here. Others are possible and may be useful, depending on circumstances. RD can be viewed as the maximum detection range of the threat by CAP aircraft, and RC as a maximum range for identifying the threat and estimating its raid size by the CAP aircraft. RL can be considered a maximum AAM launch range, RM a maximum AAM flight range, RF a maximum CAP flight distance, and RS a maximum CAP station radius.

YD, YC, YL and YM are the maximum distances of the threat from TF center at which it can be detected, CAP can be committed to engage it, AAMs can be launched, and AAM intercepts can occur (respectively). YF is a constraint that has been inserted as analogous to those just mentioned, although no physical significance has yet been attached to it. YASM is the threat's ASM Release Range. H is the minimum time for analyzing the threat and deciding to commit CAP to engage it. Constraints (14) through (17) are restrictions on the target aspect angles at detection, CAP commitment, AAM launch, and intercept (respectively). In each case, they prevent these angles from being too large. For example, constraint (14) requires that

$$-\omega_{DO} \leq \omega_D \leq \omega_{DO},$$

and so forth.

TABLE 3. FORCED CONSTRAINTS

$$\rho_D \leq R_D \tag{1}$$

$$\rho_C \leq R_C \tag{2}$$

$$\rho_L \leq R_L \tag{3}$$

$$\rho_M \leq R_M \tag{4}$$

$$\rho_F \leq R_F \tag{5}$$

$$\rho_S \leq R_S \tag{6}$$

$$T_D \leq Y_D \tag{7}$$

$$T_C \leq Y_C \tag{8}$$

$$T_L \leq Y_L \tag{9}$$

$$T_M \leq Y_M \tag{10}$$

$$T_M \geq Y_{ASM} \tag{11}$$

$$T_F \leq Y_F \tag{12}$$

$$h \geq H \tag{13}$$

$$\cos \hat{\omega}_D \geq \cos \omega_{D0} \tag{14}$$

$$\cos \omega_C \geq \cos \omega_{C0} \tag{15}$$

$$\cos \omega_L \geq \cos \omega_{L0} \tag{16}$$

$$\cos \omega_M \geq \cos \omega_{M0} \tag{17}$$

$$\cos \omega_F \geq \cos \omega_{F0} \tag{18}$$

Constraints (14) through (17) are couched in terms of limitations on the cosines of the angles, rather than in terms of the angles themselves. The reason for this will become more apparent in Section 5, where it is suggested that algebraic functions be substituted for trisonometric ones via the substitutions:

$$\cos\ (\text{angle}) = x$$
$$\sin\ (\text{angle}) = y$$

and the introduction of the following relation among these new variables:

$$x^2 + y^2 = 1$$

3.2 Natural Constraints

Those natural constraints that seem most reasonable to impose are shown in Table 4. Natural constraints (7) through (12) are stated explicitly in anticipation of the suggested change from trigonometric to algebraic functions mentioned above and discussed in Section 5.

3.3 Discussion of Constraints

Natural constraints (4) and (4a) are equivalent because VM is non-negative. Similarly, natural constraints (5) and (5a) are equivalent because VF is non-negative.

Forced constraints may be dropped qualitatively by omitting a constraint, and this is sometimes the desirable way to proceed. However, it is also sometimes desirable to drop a constraint quantitatively by setting its bound to such a large (or small) value that the constraint has no effect on the solution.

This memorandum considers only those cases in which the bounding values shown in Table 5 have fixed values given by the conditions of the problem. Except for the trigonometric functions, these values will be non-negative for most problems.

TABLE 4. NATURAL CONSTRAINTS

$$\rho_D \geq 0 \tag{1}$$

$$\rho_C \geq 0 \tag{2}$$

$$\rho_L \geq 0 \tag{3}$$

$$\rho_M \geq 0 \quad (t_M \geq 0) \qquad (4)(4a)$$

$$\rho_F \geq 0 \quad (t_F \geq 0) \qquad (5)(5a)$$

$$\rho_S \geq 0 \tag{6}$$

$$\cos^2 \omega_D + \sin^2 \omega_D = 1 \quad , \quad 0° \leq \omega_D \leq 180° \tag{7}$$

$$\cos^2 \omega_C + \sin^2 \omega_C = 1 \quad , \quad 0° \leq \omega_C \leq 180° \tag{8}$$

$$\cos^2 \omega_L + \sin^2 \omega_L = 1 \quad , \quad -180° \leq \omega_L \leq 180° \tag{9}$$

$$\cos^2 \omega_M + \sin^2 \omega_M = 1 \quad , \quad -180° \leq \omega_M \leq 180° \tag{10}$$

$$\cos^2 \omega_F + \sin^2 \omega_F = 1 \quad , \quad -180° \leq \omega_F \leq 180° \tag{11}$$

$$\cos^2 \alpha + \sin^2 \alpha = 1 \quad , \quad 0° \leq \alpha \leq 180° \tag{12}$$

TABLE 5. LIMITING VALUES

R_D	H
R_C	$\cos \omega_{D0}$
R_L	$\cos \omega_{C0}$
R_M	$\cos \omega_{L0}$
R_F	$\cos \omega_{M0}$
R_S	$\cos \omega_{F0}$
Y_D	
Y_C	
Y_L	
Y_M	
Y_{ASM}	
Y_F	

4. OBJECTIVE FUNCTIONS

Table 6 shows some possible objectives to be achieved by manipulating the controllable variables. A choice of one of them (or of another not shown in Table 6) depends on a mixture of considerations involving mathematical convenience and appropriateness to the applications being contemplated for the solution.

Objectives (1) and (2) are equivalent, and both ask for the largest angle subtended by the region defended by the CAP station. Objective (3) asks for the maximum arc length, and objective (4) for the maximum length of "front," defended by the CAP station. Objectives (5) and (6) ask for the maximum-area circular sector and triangle defended by the CAP station. Objective (7) asks for the maximum time delay attainable -- if this objective is adopted it is advisable to add a forced constraint preventing alpha from being too small, since otherwise the maximum h will probably occur at alpha = 0. Such a bound on alpha is also advisable if objective (8), i.e., maximizing the distance between the intercept point and TF center, is selected. Objectives (9), (10) and (11) are variations of objective (8) that automatically prevent the optimizing alpha from becoming zero. If objective (12) is adopted, then it may be advisable to add a forced constraint that prevents rhoS from becoming too small, since otherwise the minimum may occur at rhoS = 0.

TABLE 6. CANDIDATE OBJECTIVES

Maximize α	(1)
Minimize $\cos \alpha$	(2)
Maximize $\alpha\rho_S$	(3)
Maximize $\rho_S \sin \alpha$	(4)
Maximize $\alpha\rho_S^2$	(5)
Maximize $\rho_S^2 \sin \alpha \cos = \frac{1}{2} \rho_S^2 \sin (2\alpha)$	(6)
Maximize h	(7)
Maximize Y_M	(8)
Maximize $Y_M\ \alpha$	(9)
Maximize $Y_M^{\ 2}\ \alpha$	(10)
Maximize $Y_C^{\ 2}\ \alpha$	(11)
Minimize $\rho_S \cos \alpha$	(12)

5. DISCUSSION

If the CAP aircraft are not equipped with long-range AAMs, then take RM = 0. By forced constraint (4), this will immediately imply rhoM = 0 and hence by relation (16) of Table 1 tM = 0. Then it follows from relations (13) through (15) that TL = TM = TF. And then rhoL*sin(omegaL) = rhoL*cos(omegaL) = 0; hence rhoL = 0.

It is strongly suggested that before a nonlinear optimization code is applied to this problem, all trigonometric functions be replaced by their component coordinate values and appropriate additional constraints be introduced to express the relation among the component coordinate values. Thus, for example, we urge the replacements:

$$\cos(\text{alpha}) = AX,$$
$$\sin(\text{alpha}) = AY,$$
$$AX^2 + AY^2 = 1,$$

and similarly for all the other angles. The aim here is to rid the problem of trigonometric functions, replacing them with expressions that are no worse than quadratic in the variables. When this is done, all the necessary numerical evaluations of functions and their gradients can be done by multiplication and addition, which are performed much quicker than the evaluation of trigonometric functions.

It is sometimes hard to find an initial feasible solution, especially to nonlinear problems such as this one. Nevertheless, in this case, an initial feasible solution can conveniently be found by the following argument. First, it seems intuitively obvious that there is a feasible solution for the original nonlinear problem if and only if there is a feasible solution for the "collinearized" problem obtained by setting all of the angle variables to zero. It is easily seen that all the relations among variables and constraints for such a "collinearized" problem are either linear equations or linear inequalities. So if we adopt them as the constraints and ask for a maximum value of rhoS, we find (see Table 7) that we have a linear programming problem, the solution of which (if there is one) provides an initial feasible solution for the original nonlinear programming problem.

Various search, quasilinearization, or feasible direction methods can be used to improve on any current feasible (but nonoptimal) solution. See reference (d) for a survey of potentially applicable techniques and references to other sources. If this work is to be pursued, it is recommended that either a Lagrange multiplier or feasible direction method be tried.

TABLE 7. COLLINEARIZED PROGRAMMING PROBLEM

Maximize: ρ_S

Subject to:

$$\rho_D = T_D - \rho_S$$

$$\rho_C = T_C - \rho_S$$

$$\rho_L + \rho_F = T_L - \rho_S$$

$$\rho_M + \rho_F = T_M - \rho_S$$

$$\rho_F = T_F - \rho_S$$

$$T_C = T_D - V_T h$$

$$T_L = T_C - V_T t_F$$

$$T_M = T_L - V_T t_M$$

$$T_F = T_M - \rho_M$$

$$\rho_F = V_F t_F$$

$$\rho_M = V_M t_M$$

Table 3, Forced Constraints (1) through (13) apply.

Table 4, Natural Constraints (1) through (6) apply.

REFERENCES

(a) NAVAIRDEVCEN Systems Directorate Tech Memo No. SD TM-25-81 "CAP Stationing Analysis" of 17 Jul 1981

(b) NAVAIRDEVCEN Systems Directorate Tech Memo No. SD TM 20-81 "CAPS (CAP Stationing) Computer Program" of 1 Jul 1981

(c) NALCON Newsletter No. 5 "CONMIN: A Program for Solution of Linear and Nonlinear Constrained Otpimization Problems" of Feb 1981

(d) Mordecai Avriel "Nonlinear Programs: Analysis and Methods," Prentice-Hall, Englewood Cliffs, NJ 1976